# MODELLING DYNAMIC TRAFFIC FLOW IN NETWORKS

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***Abstract***

In this paper a simulation model for time dependent traffic flow in networks with multiple sources and sinks is presented. The main goals are the real-time simulation of large highway networks and the development of a planning tool for traffic networks. First, the assumptions beyond the model are presented. Second, I describe the adaption of the model to numerical simulations and the constructions of networks. The next sections contain a short note on decision processes and our approach to this field. Then an extension of the model to a larger framework, and possible fields of application are proposed. In the last section a simulation of the traffic flow in a network is shown.

**Key-Words**

Dynamic Traffic Flow - Dynamic Traffic Networks - Macroscopic simulation model - Non linear Dynamics

## 1. Introduction

In recent years the number of traffic jams and breakdowns on the highways have increased significantly. This is due to the fact of a steady increase of the daily traffic volume, while the capacity of the traffic network was kept nearly constant. In Germany the extension of the highway network has come to an end for several, financial and ecological, reasons. Therefore the need of the optimization of the traffic flow on the existing network, together with other measurements, is obvious. In this paper a phenomenological simulation model of the traffic flow in networks is presented. The need for such models is evident : It is impossible to control (and within this to optimize) the traffic flow in a network without any knowledge about the future evolution of the dynamic load in the observed network. Therefore we need models that allow for the prediction of the state of the network faster than in real time.

## 2. Idea of the model

The traffic flow on highway networks consists of many cars in each of them sitting a driver with an individual behavior. This simple statement immediately leads to one of the main problems in modelling the traffic flow on a road : If we want to take into account the individual (microscopic) behavior of each driver, we must process an enormous mass of information to obtain the systems behavior. There are models dealing with this approach (e.g. [13] {Wiedemann}), but even with the fastest computers it is not possible to simulate the traffic flow in a larger network in real-time.

Therefore the main idea is to neglect the individual (microscopic) behaviorof the drivers and to develop a model for a macroscopic description of the evolution of the traffic flow, dealing with statistical quantities, that can be derived by aggregating the quantities on the microscopic level.

At once the question arises : Are there related fields of research with comparable problems ?

The answer is : Yes, hydrodynamics succeed in describing the macroscopic flow of fluids too, without any knowledge about the actual individual behavior of the particles. Therefore hydrodynamics may give us a framework, that simplifies the construction of a model significantly. Nevertheless for the adaption of the theory of hydrodynamics we have to modify the equations in a reasonable way. Therefore we have to think about the differences between the flow of a fluid and the traffic flow on a road.

## 3. Assumptions beyond the model

The first hydrodynamical models for the description of traffic flow weredeveloped in the fifties by Lighthill and Whitham [3] and Richards [10]. Since then various people worked on that field and improved the models in several ways (e.g. [1], [4], [5], [7], [12]). Our concern in this section is to give a short review about the properties with regard to the adaption for numerical simulations. A more comprehensive discussion of the model is given in [2].

First we have to list the similarities between a fluid and the traffic flow :

– The density $r(x,t)$ of a fluid corresponds to the number of cars on an given length of a road.

– The velocity of the particles in the fluid corresponds to the velocity of the cars.

– In both systems holds a conservation law. This means, that in a system without any sources and sinks the number of particles, as well as the number of cars must remain constant. Mathematically this is expressed with the so-called *continuity equation* :

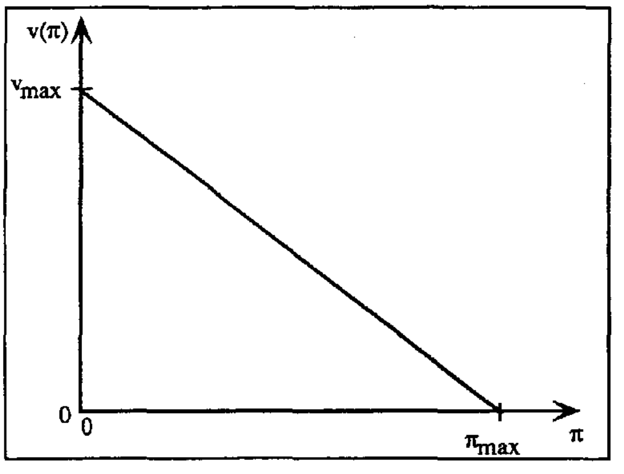
$$\frac{\partial \pi(x,t)}{\partial t} + \frac{\partial j(x,t)}{\partial x} = 0 \qquad (1)$$

where $r(x,t)$ is the density of the fluid (cars) at place $x$ at time $t$, while $j(x,t)$ is corresponding flow.

Now we have to consider the differences between these two systems :

– The hydrodynamical flow $j(x,t)$ is defined as $j(x,t) = r(x,t) \cdot v(x,t)$. This is different to the behavior of the drivers. They take into account the velocity of the cars $v(x+s,t)$ a certain distance $s$ ahead. Therefore, in this model, the phenomenological ansatz for the flow reads as $j(x,t) = r(x,t) \cdot v(x+s,t)$.

– In contrast to the behavior of a fluid, theoretical considerations (e.g. Prigogine [8]) as well as empirical investigations show a functional relationship between the density of cars and their average stationary velocity $v(x,t)=v(r(x,t))$. The most simple relationship is given in the following picture :




– In traffic flow after a sudden change of the density, due to the reaction time of the drivers and cars, the actual velocity will not reach the «stationary velocity» instantaneously. This behavior is described by a relaxation ansatz :

$$\frac{\partial v(x,t)}{\partial t} = \frac{1}{\tau} \left[v(\rho(x,t)) - v(x,t)\right] \qquad (2)$$

## 4. Adaption for numerical simulations

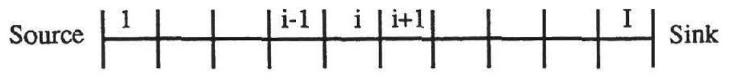
The task in this section is to construct a simulation model, that includes the properties of the traffic flow given above, and that can be performed on a computer easily.

– First, for computational reasons, the roads on the network have to be split up into cells $i = 1,\ldots,I$, and we have to give a differential equation for the evolution of the density in each cell. This is a discrete form of the *continuity equation* :

$$\frac{dN(i,t)}{dt} = \Delta x_i \frac{\delta\rho(i,t)}{\partial t} = j(i-1 \longrightarrow i,t) - j(i \longrightarrow i+1,t) \quad (3)$$

– Second we have to include the foresight of the drivers. In accordance to the flow given in the preceding section we write :

$$j(i \longrightarrow i+1,t) = r(i,t) \cdot v(i+1,t) \quad (4)$$

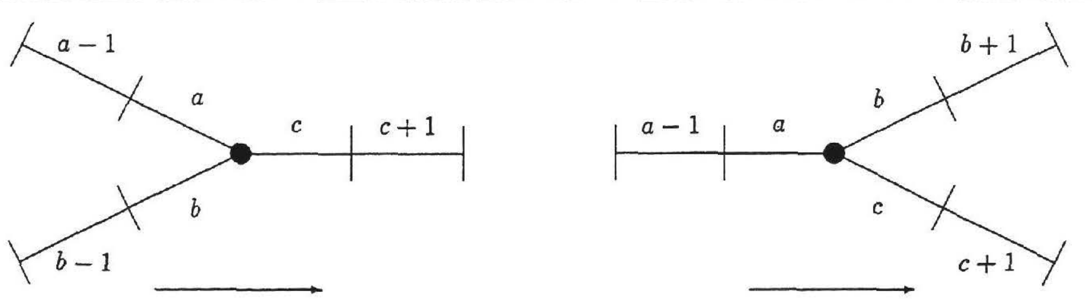



– Last we need an additional differential equation for the velocity for every cell of the road :

$$\frac{\partial v(i,t)}{\partial t} = \frac{1}{\tau}\,[v(\rho(i,t)) - v(i,t)] \quad (5)$$

## 5. Construction of networks

The discrete model, given in the section above, allows for the construction of complex networks. An arbitrary node (intersection) can be constructed as a combination of two basic items :




Converging of two links into one — Diverging of one link into two

– Two links (roads) converge to one link, and

– One link (road) diverges to two links.

Within the frame work of the model the converging of two links into one (picture to the left) is given as :

$$\begin{aligned}
\Delta x_a \frac{\partial\rho(a,t)}{\partial t} &= \rho(a-1,t)v(a,t) - \rho(a,t)v(c,t) \\
\Delta x_b \frac{\partial\rho(b,t)}{\partial t} &= \rho(b-1,t)v(b,t) - \rho(b,t)v(c,t) \qquad (6) \\
\Delta x_i \frac{\partial\rho(c,t)}{\partial t} &= \{\rho(a,t) + \rho(b,t)\}\,v(c,t) - \rho(b,t)v(c+1,t)
\end{aligned}$$

The flow out the cells $a$ and $b$ equals the flow into cell $c$. So flow conservation is still guaranteed.

The diverging of a link into two links (picture to the right) is modelled in a similar way, but, in addition, the wishes of the drivers have to be taken into consideration.

This is done by multiplying the flow out of cell $a$ with time dependent functions $p(a,b,t)$, $p(a,c,t)$, which describe the probability of a driver coming from $a$, to select a certain route $b$ resp. $c$. Naturally the probabilities depend on the destination of the drivers, too. Therefore one has to distinguish drivers to different destinations.

Neglecting the complication of different destinations this is expressed as :

$$
\begin{aligned}
\Delta x_a \frac{\partial \rho(a,t)}{\partial t} &= \rho(a\text{-}1,t)v(a,t) - \rho(a,t)\{v(b,t)p(a,b,t) + v(c,t)p(a,c,t)\} \\
\Delta x_b \frac{\partial \rho(b,t)}{\partial t} &= \rho(a,t)v(b,t)p(a,b,t) - \rho(b,t)v(b+1,t) \\
\Delta x_i \frac{\partial \rho(c,t)}{\partial t} &= \rho(a,t)v(c,t)p(a,c,t) - \rho(c,t)v(c+1,t)
\end{aligned}
\qquad (7)
$$

The probabilities $p$ can be described by various route choice models.

## 6. Note on decision processes

As mentioned in the section above the probability of a driver to chose a certain route depends on many factors. Some of these are for example :

– destination of the driver (e.g. length of a certain route, travelling time for a certain route),

– informations, of the driver (e.g. about traffic jams on a route) and

– personal preferences of the driver.

This means, that every driver has a discrete set of alternatives (discrete set of routes), where he has to chose. Each alternative is chosen with a different probability. These probabilities can be described with so-called «Discrete Choice Models». Some classical models are : the multinomial logit model, the multinomial probit model and the nested logit model. Besides of these models in our institute the so called master equation approach was developed.

All these models introduce a so-called «utility function», that measures the utility for a driver to choose a certain alternative. The utility for an alternative depends on the characteristics of this alternative, and, in the master equation approach, on the actual state of the system. Besides of the master equation approach a survey of these models is given in [6]. An introduction to the master equation approach is given e.g. in [11].

## 7. The master equation approach

The master equation describes the evolution of a probability distribution $P(\mathbf{n},t)$ starting from an initial distribution $P(\mathbf{n},0)$, if the transition rates of the system $w_t(\mathbf{n},\mathbf{n}')$ are given. The master equation reads as :

where

$\mathbf{n}$ is the state of the system.

$P(\mathbf{n},t)$ is the probability to find the system in state $\mathbf{n}$ **at time** $t$.

$w_t(\mathbf{n},\mathbf{n}')$ is the transition probability per time unit from state $\mathbf{n}$ to state $\mathbf{n}'$, given, that $\mathbf{n}$ **is realized at time** $t$. This depends on the system's state itself as well as on external influences and time.

A possible form of the transition rates $w_t(\mathbf{n},\mathbf{n}')$ reads :

$$w_t(\mathbf{n},\mathbf{n}') = \nu_{\mathbf{n}',\mathbf{n}} \cdot exp[u_{\mathbf{n}'}(t) - u_{\mathbf{n}}(t)]$$

where

$\nu_{\mathbf{n}',\mathbf{n}}$ is a flexibility parameter.

$u_{\mathbf{n}'}(t)$ is a dynamical utility function.

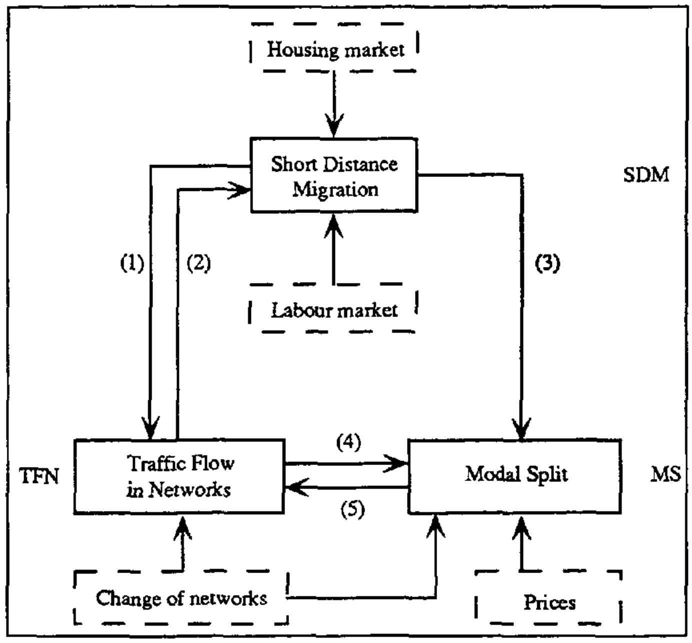
The dynamical utility function \$u\$ depends on external parameters and the systems state itself. The external parameters include the (time dependent) route characteristics (e.g. route length, travelling time, occurrence of traffic jams).

In contrast to other discrete choice models the master equation approach allows for the computation of the evolution to the stationary state of the system. Other models just consider the stationary state of the system without any possibilities to calculate the evolution.

## 8. Extension to a larger framework

Within the the master equation approach the given model can be seen as a short term module of a larger framework for an integrated description of transportation and settlement activities in a region. We suggest as other modules in that framework the modal split and the short distance migration. A more comprehensive description is given in [9]. The diagram given below gives an illustration about our ideas.

(1) e.g. origin-destination relations, traffic volume
(2) e.g. travel time, costs
(3) e.g. traffic volume
(4) e.g. trip time, trip costs
(5) e.g. partial volumes of transport




## 9. Possible results and fields of application

There are several results, that can be obtained from the presented model. First of all it allows for a forecast of the evolution of the traffic in a network. This gives a possibility to develop controlling strategies to optimize the flow. Moreover, the model gives the possibility to calculate the travelling times and delay times in a network. In combination with these results the consumption of fuel and the pollution of the air, caused by the traffic can be calculated.

Besides of the inclusion of the model in the integrated framework, there are two main fields of application of the model. On one hand the model gives the possibility for a dynamic control of the traffic flow in a network, because it allows for the real time calculation of the efficiency of e.g. time dependent limits for the velocity or route recommendations to avoid congestion and traffic jams. On the other hand the model gives a tool for testing the efficiency of changes of the topology in an existing network (e.g. construction of new roads). Moreover, it is an useful tool for a comparison of the properties of different network topologies.

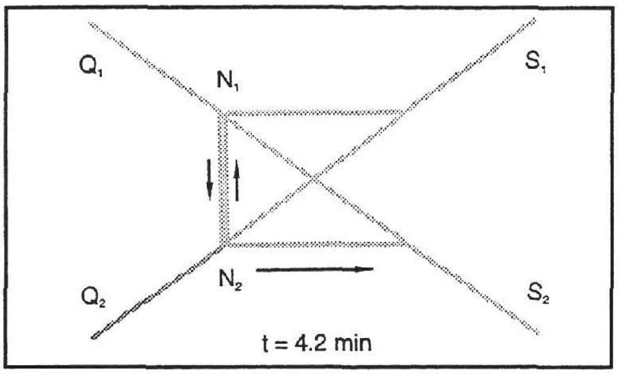
## 10. Simulation

The next six pictures show a simulation of a small traffic network. The given network has two sources $Q_1$ and $Q_2$ on the left side. Two sinks $(S_1,S_2)$ are placed on the right side of the network. From each source a part of the drivers is destinated to sink $S_1$, the other part to sink $S_2$. Therefore, the drivers have to decide for their route at the nodes $N_1$ or $N_2$. In this simulation the multinomial logit model is used to determine the probability of choosing a certain route only depending on the length of the route. The network consists of 320 cells with a length of 200 m, yielding a total length of 64 km.

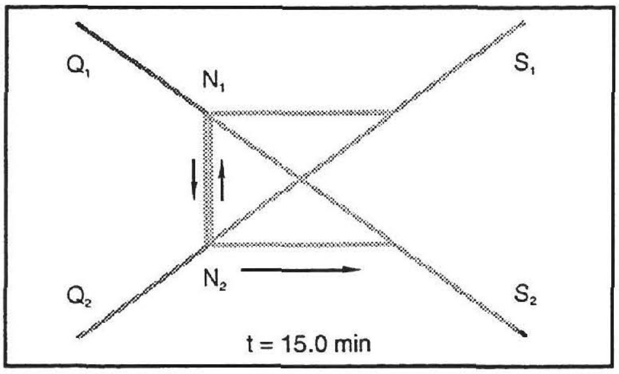
The arrows in in the following figures indicate the direction of the traffic flow. There is only one-way traffic on every link, except the two links between the nodes $N_1$ and $N_2$ (indicated by the two short arrows). Two-way traffic is modelled with two different links in opposite directions. There is no interdependence between the two links, but can be modelled as well.

In the following pictures the density in the network is indicated by the grey level. The darker the grey level, the higher the density. An offset is used to denote the empty network.

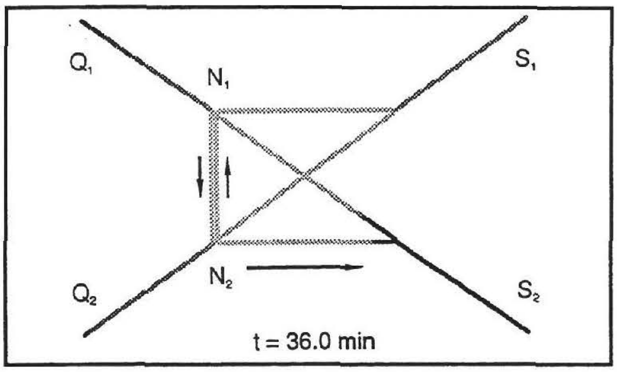
The simulation starts with an empty network at $t=0.0$. The parameters of the cells are : $r_{max} = 60$ cars/km, $v_{max} = 120$ km/h, t = 0sec (this is an slightly simplified model, that gives the effective flow in the network) and D$x$ = 0.2 km. The linear v($r$)–relationship was used.



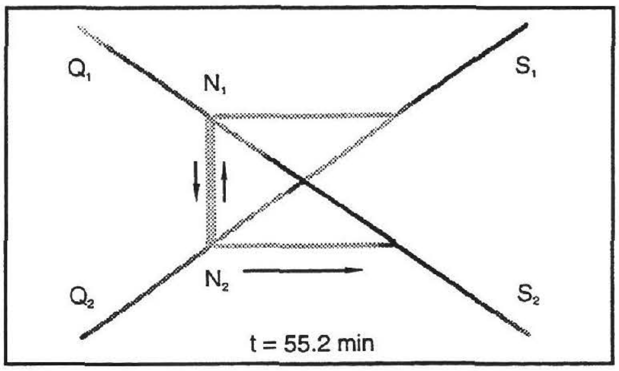

The network is empty, besides of the first cars entering the network from $Q_2$. There is still no flow into the network coming from $Q_1$.



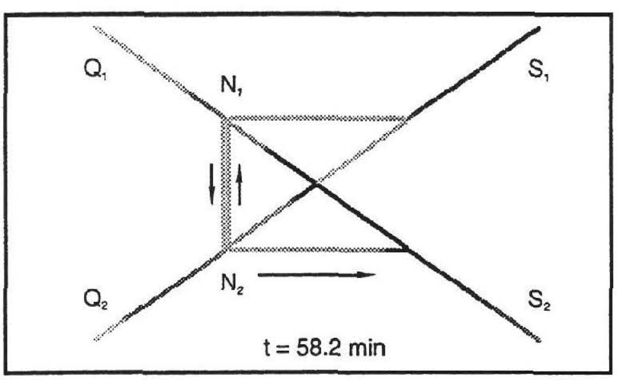

Now the inflow from $Q_2$ into the network has stopped, too. Despite of the opening of $S_2$ the the total length of the traffic jam in front of $S_2$ is sill increasing. Moreover, looking at the densities belonging to a certain sink (not shown here), even cars destinated to $S_1$ have to wait in the traffic jam until they can pass the node in the middle of the network. This is the reason why the link from the node in the middle to $S_1$ is practically empty.



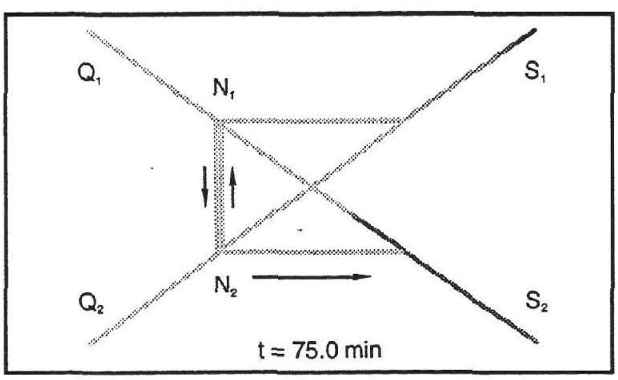

In this picture, there is a long traffic jam in front of $S_2$., even continuing in the preceding links. There is a slightly higher density in front of $S_1$. Cars are entering the network from both sources $Q_1$ and $Q_2$.




The traffic jam in front of $S_2$. has grown even more, but the velocity in sink $S_2$. has been increased. The density just in front of $S_2$. has lowered down. Now there is a significantly higher density in front of $S_1$. As can be seen, no more cars entering the network at $Q_1$.




Now cars are entering the network from source $Q_1$, too. As can be seen, the first cars reached sink $S_2$.. This corresponds to the maximum speed of 120 km/h, that gives a minimum travelling time of 9 min for the shortest route (A delay until the traffic jam occurs must be added). This traffic jam occurs due to the low velocity at $S_2$.




Now the traffic jams are disappearing. The last cars are leaving the network.

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